

Find the Fourier series of the following function $f(x) = x + \pi$, if $-\pi < x < \pi$, and $f(x + 2\pi) = f(x)$.

[106 台大化工 1]

[解]令 $F(x) = f(x) - \pi$ ，則 $F(x)$ 為奇函數，其 Fourier 級數為

$$F(x) = \sum_{n=1}^{\infty} b_n \sin nx, \text{ 其中}$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} x \sin nxdx = \frac{2}{\pi} \left(-\frac{1}{n} \right) \left(x \cos nx \Big|_0^{\pi} - \int_0^{\pi} \cos nxdx \right)$$
$$= -\frac{2}{n\pi} (\pi \cos n\pi) = -\frac{2}{n} (-1)^n$$

$$F(x) = -2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin nx \Rightarrow f(x) = \pi - 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin nx$$

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