Transform the function $f(x)=\begin{cases} x, & \text{if } 0<x<a \\ 0, & \text{if } x>a \end{cases}$ in the form of sine integral. [105 中正光電 6]

[解] $F_S(\omega)=\int_0^\infty f(x)\sin\omega x dx=\int_0^a x\sin\omega x dx=-\frac{1}{\omega}(x\cos\omega x\Big|_0^a-\int_0^a\cos\omega x dx)$

$$=-\frac{1}{\omega}(a\cos\omega a-\frac{\sin\omega x}{\omega}\Big|_0^a)=-\frac{1}{\omega}(a\cos\omega a-\frac{\sin\omega a}{\omega})=\frac{\sin\omega a-\omega a\cos\omega a}{\omega^2}$$

$$f(x)=\frac{2}{\pi}\int_0^\infty\frac{\sin\omega a-\omega a\cos\omega a}{\omega^2}\sin\omega x d\omega$$