Find the Fourier transform of the function $f(x)=xe^{-x^2}$. [105 中山光電 5]

[解] $\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx = \int_{-\infty}^{\infty} xe^{-x^2}e^{-i\omega x}dx = \int_{-\infty}^{\infty} xe^{-(x^2+i\omega x)}dx = \int_{-\infty}^{\infty} xe^{-(x+\frac{i\omega}{2})^2-\frac{\omega^2}{4}}dx = e^{-\frac{\omega^2}{4}}\int_{-\infty}^{\infty} xe^{-(x+\frac{i\omega}{2})^2}dx$

$$= e^{-\frac{\omega^2}{4}}\int_{-\infty}^{\infty}[(x+\frac{i\omega}{2})-\frac{i\omega}{2}]e^{-(x+\frac{i\omega}{2})^2}dx$$

$$= e^{-\frac{\omega^2}{4}}[\int_{-\infty}^{\infty}(x+\frac{i\omega}{2})e^{-(x+\frac{i\omega}{2})^2}dx-\int_{-\infty}^{\infty}\frac{i\omega}{2}e^{-(x+\frac{i\omega}{2})^2}dx]$$

$$= e^{-\frac{\omega^2}{4}}[\frac{1}{2}\int_{-\infty}^{\infty}e^{-(x+\frac{i\omega}{2})^2}d(x+\frac{i\omega}{2})^2-\int_{-\infty}^{\infty}\frac{i\omega}{2}e^{-(x+\frac{i\omega}{2})^2}d(x+\frac{i\omega}{2})]$$

$$= e^{-\frac{\omega^2}{4}}[-\frac{1}{2}e^{-(x+\frac{i\omega}{2})^2}\Big|_{-\infty}^{\infty}-\frac{i\omega}{2}\sqrt{\pi}] = -\frac{i\omega}{2}\sqrt{\pi}e^{-\frac{\omega^2}{4}}$$