

試求函數 $f(x)$ 的傅立葉餘弦級數(Fourier cosine series)。 $f(x)=x, 0 < x < \pi$ 。 [104高第一環安7]

[解]設 $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x dx = \frac{2}{\pi} \cdot \frac{x^2}{2} \Big|_0^{\pi} = \frac{2}{\pi} \cdot \frac{\pi^2}{2} = \pi$$

$$\begin{aligned} a_n &= \frac{2}{\pi} \int_0^{\pi} x \cos nx dx = \frac{2}{\pi} \cdot \frac{1}{n} (x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nxdx) = \frac{2}{n\pi} (0 + \frac{\cos nx}{n} \Big|_0^{\pi}) \\ &= \frac{2}{n^2\pi} (\cos n\pi - 1) = \frac{2}{n^2\pi} [(-1)^n - 1] = -\frac{4}{(2n-1)^2\pi} \end{aligned}$$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos(2n-1)x$$