Find the Fourier integral representation of the following non-periodic function:

$$f(\theta)=\begin{cases}\cos\theta, & -\frac{\pi}{2}<x<\frac{\pi}{2}\\ 0, & \text{otherwise}\end{cases}$$. [104 高師大電子 5]

[解]令 $f(\theta)=\int_0^\infty [a(\omega)\cos\omega\theta+b(\omega)\sin\omega\theta]d\omega$，$f(\theta)$為偶函數 $\Rightarrow b(\omega)=0$

$$a(\omega)=\frac{2}{\pi}\int_0^\infty f(\theta)\cos\omega\theta d\theta=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\cos\theta\cos\omega\theta d\theta$$

$$=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\cos(1+\omega)\theta+\cos(1-\omega)\theta d\theta=\frac{1}{\pi}\cdot[\frac{\sin(1+\omega)\theta}{1+\omega}+\frac{\sin(1-\omega)\theta}{1-\omega}]\Big|_0^{\frac{\pi}{2}}$$

$$=\frac{1}{\pi}[\frac{\sin\frac{(1+\omega)\pi}{2}}{1+\omega}+\frac{\sin\frac{(1-\omega)\pi}{2}}{1-\omega}]=\frac{1}{\pi}\frac{(1-\omega)\sin\frac{(1+\omega)\pi}{2}+(1+\omega)\sin\frac{(1-\omega)\pi}{2}}{1-\omega^2}$$

$$=\frac{1}{\pi}\frac{[\sin\frac{(1+\omega)\pi}{2}+\sin\frac{(1-\omega)\pi}{2}]-\omega[\sin\frac{(1+\omega)\pi}{2}-\sin\frac{(1-\omega)\pi}{2}]}{1-\omega^2}$$

$$=\frac{1}{\pi}\frac{2\sin\frac{\pi}{2}\cos\frac{\omega\pi}{2}-2\omega\cos\frac{\pi}{2}\sin\frac{\omega\pi}{2}}{1-\omega^2}=\frac{1}{\pi}\frac{2\cdot1\cdot\cos\frac{\omega\pi}{2}-2\omega\cdot0\cdot\sin\frac{\omega\pi}{2}}{1-\omega^2}=\frac{2\cos\frac{\omega\pi}{2}}{\pi(1-\omega^2)}$$

$$f(\theta)=\frac{2}{\pi}\int_0^\infty\frac{\cos\frac{\omega\pi}{2}}{1-\omega^2}\cos\omega\theta d\omega$$