Find the complex Fourier series of the $f(x)$ on the given interval. [104 清大生醫丙 8]

$$f(x) = \begin{cases} 0, & -\dfrac{1}{2} < x < -\dfrac{1}{4} \\[2mm] 1, & -\dfrac{1}{4} < x < \dfrac{1}{4} \\[2mm] 0, & \dfrac{1}{4} < x < \dfrac{1}{2} \end{cases}$$

[解]令 $f(x) = \displaystyle\sum_{n=-\infty}^{\infty} c_n e^{i2n\pi x}$, $\quad -\dfrac{1}{2} < x < \dfrac{1}{2}$

$$c_n = 2\int_{-1/2}^{1/2} f(x)e^{-i2n\pi x}dx = 2\int_{-1/4}^{1/4} 1 \cdot e^{-i2n\pi x}dx = \frac{1}{-in\pi}e^{-i2n\pi x}\Big|_{-1/4}^{1/4}$$

$$= \frac{1}{-in\pi}(e^{-in\pi/2} - e^{in\pi/2}) = \frac{1}{in\pi} \cdot 2\sinh\frac{n\pi}{2}$$

$$\therefore f(x) = \frac{2}{i\pi}\sum_{n=-\infty}^{\infty}\frac{1}{n}\sinh\frac{n\pi}{2}e^{i2n\pi x}, \quad -\frac{1}{2} < x < \frac{1}{2}$$