

Find the Fourier cosine transformation of  $f(x) = e^{-x}$ . [103 高海電訊 4]

$$\begin{aligned} \text{[解]先推導} \int e^{ax} e^{ibx} dx &= \int e^{(a+ib)x} dx = \frac{e^{(a+ib)x}}{a+ib} = \frac{(a-ib)e^{(a+ib)x}}{a^2+b^2} = \frac{e^{ax}(a-ib)(\cos bx + i \sin bx)}{a^2+b^2} \\ &= \frac{e^{ax}[(a \cos bx + b \sin bx) + i(a \sin bx - b \cos bx)]}{a^2+b^2} \end{aligned}$$

$$\text{實部} \int e^{ax} \cos bxdx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2+b^2}, \text{虛部} \int e^{ax} \sin bxdx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2+b^2}$$

$f(x)$ 的 Fourier cosine 轉換為

$$F_c(\omega) = \int_0^{\infty} f(x) \cos \omega x dx = \int_0^{\infty} e^{-x} \cos \omega x dx = \frac{e^{-x}(-\cos \omega x + \omega \sin \omega x)}{(-1)^2 + \omega^2} \Big|_0^{\infty}$$

$$= 0 - \frac{1 \cdot (-1 + 0)}{1 + \omega^2} = \frac{1}{1 + \omega^2}$$