Expand the given function in an appropriate cosine or sine Fourier series. $f(x)=|\sin x|,\ -\pi<x<\pi$. [103逢甲電機4]

[解] $f(x)$為偶函數，其Fourier級數為

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx$$

$$a_0=\frac{2}{\pi}\int_0^{\pi}(-x+\pi)dx=\frac{2}{\pi}\cdot(-\frac{x^2}{2}+\pi x)\Big|_0^{\pi}=\frac{2}{\pi}\cdot(-\frac{\pi^2}{2}+\pi^2)=\pi$$

$$a_n=\frac{2}{\pi}\int_0^{\pi}(-x+\pi)\cos nxdx=\frac{2}{\pi}(\frac{1}{n})\left((-x+\pi)\sin nx\Big|_0^{\pi}+\int_0^{\pi}\sin nxdx\right)$$

$$=\frac{2}{n\pi}(0-\frac{\cos nx}{n}\Big|_0^{\pi})=-\frac{2}{n^2\pi}(\cos n\pi-1)=-\frac{2}{n^2\pi}[(-1)^n-1]=\frac{4}{(2n-1)^2\pi}$$

$$f(x)=\frac{\pi}{2}+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)x$$