(a)Find the Fourier transform of $x(t)=\begin{cases}1, & |t|<1\\ 0, & |t|>1\end{cases}$. (b)Use (a) result to calculate $\int_{-\infty}^{\infty}\frac{\sin^2\omega}{\omega^2}d\omega$.

(Hint: Parseval's relation: $\int_{-\infty}^{\infty}|x(t)|^2\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega$). [103 清大生醫甲 2]

[解](a) $X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-i\omega t}dt=\int_{-1}^{1}e^{-i\omega t}dt=\frac{e^{-i\omega}-e^{i\omega}}{-i\omega}=\frac{2\sin\omega}{\omega}$

(b)由Parseval's關係式得

$$\int_{-\infty}^{\infty}|x(t)|^2\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega \Rightarrow \int_{-1}^{1}1^2\cdot dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{2\sin\omega}{\omega}\right)^2 d\omega$$

$$2=\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{\sin^2\omega}{\omega^2}d\omega \Rightarrow \int_{-\infty}^{\infty}\frac{\sin^2\omega}{\omega^2}d\omega=\pi$$