There is periodic square wave with analytic with represented as $f(x)$ function

$f(x) = \begin{cases} -k, & \text{when } -\pi < x < 0 \\ k, & \text{when } 0 < x < \pi \end{cases}$, and $f(x+2\pi) = f(x)$. Please find the Fourier coefficients a_n, b_n and

their series functions to represent the $f(x)$ functions. [103元智機械7]

[解]$f(x)$為奇函數 $\Rightarrow$ 設 $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$

$b_n = \frac{2}{\pi} \int_0^{\pi} k \sin nx\, dx = -\frac{2k}{n\pi} \cdot \cos nx \Big|_0^{\pi} = -\frac{2k}{n\pi} \cdot (\cos n\pi - 1) = -\frac{2k}{n\pi} \cdot [(-1)^n - 1] = \frac{4k}{(2n-1)\pi}$

$f(x) = \frac{4k}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)x$