Find the Fourier series of the given function $f(x)$, which is assumed to have the period 2π.

$$f(x)=\begin{cases} x+\pi, & \text{if } -\pi<x<0 \\ -x+\pi, & \text{if } 0<x<\pi \end{cases}.$$ [103中山材光6]

[解] $f(x)$為偶函數，其Fourier級數為

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx$$

$$a_0=\frac{2}{\pi}\int_0^{\pi}(-x+\pi)dx=\frac{2}{\pi}\cdot\left(-\frac{x^2}{2}+\pi x\right)\Big|_0^{\pi}=\frac{2}{\pi}\cdot\left(-\frac{\pi^2}{2}+\pi^2\right)=\pi$$

$$a_n=\frac{2}{\pi}\int_0^{\pi}(-x+\pi)\cos nxdx=\frac{2}{\pi}\left(\frac{1}{n}\right)\left((-x+\pi)\sin nx\Big|_0^{\pi}+\int_0^{\pi}\sin nxdx\right)$$

$$=\frac{2}{n\pi}\left(0-\frac{\cos nx}{n}\Big|_0^{\pi}\right)=-\frac{2}{n^2\pi}(\cos n\pi-1)=-\frac{2}{n^2\pi}[(-1)^n-1]=\frac{4}{(2n-1)^2\pi}$$

$$f(x)=\frac{\pi}{2}+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)x$$