$f(x)=x^2,\ 0<x<2\pi,\ f(x)=f(x+2\pi)$. Find the Fourier series. [103 中央機械 6(a)]

[解] $f(x)=\dfrac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$,

$$a_0=\frac{1}{\pi}\int_0^{2\pi}x^2dx=\frac{1}{\pi}\cdot\frac{x^3}{3}\bigg|_0^{2\pi}=\frac{8\pi^2}{3}$$

$$a_n=\frac{1}{\pi}\int_0^{2\pi}x^2\cos nxdx=\frac{1}{n\pi}(x^2\sin nx\Big|_0^{2\pi}-\int_0^{2\pi}2x\sin nxdx)$$

$$=\frac{2}{n^2\pi}(x\cos nx\Big|_0^{2\pi}-\int_0^{2\pi}\cos nxdx)=\frac{2}{n^2\pi}(2\pi)=\frac{4}{n^2}$$

$$b_n=\frac{1}{\pi}\int_0^{2\pi}x^2\sin nxdx=-\frac{1}{n\pi}(x^2\cos nx\Big|_0^{2\pi}-\int_0^{2\pi}2x\cos nxdx)$$

$$=-\frac{1}{n\pi}[4\pi^2-\frac{2}{n}(x\sin nx\Big|_0^{2\pi}-\int_0^{2\pi}\sin nxdx)]=-\frac{1}{n\pi}(4\pi^2+\frac{2}{n^2}\cdot\cos nx\Big|_0^{2\pi})=-\frac{4\pi}{n}$$

$$f(x)=\frac{4\pi^2}{3}+4\sum_{n=1}^{\infty}(\frac{\cos nx}{n^2}-\frac{\pi\sin nx}{n})$$