

若 $f(x)$ 的傅立葉級數為 $a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x) + b_n \sin(n\pi x)$ ，試求 $f(x) = \begin{cases} x+1, & -1 < x < 0 \\ x-1, & 0 \leq x < 1 \end{cases}$ ，

且 $f(x) = f(x+2)$ 之 (a) a_0 及 a_n ，(b) b_n 。[102 虎尾電機 2]

[解] $f(x)$ 為奇函數 $\Rightarrow a_0 = 0, a_n = 0, f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x$

$$\begin{aligned} b_n &= 2 \int_0^1 f(x) \sin n\pi x dx = 2 \int_0^1 (x-1) \sin n\pi x dx = -\frac{2}{n\pi} [(x-1) \cos n\pi x]_0^1 - \int_0^1 \cos n\pi x dx \\ &= -\frac{2}{n\pi} (0+1) = -\frac{2}{n\pi} \end{aligned}$$

南臺科技大學

Southern Taiwan University of Science and Technology