Find the Fourier cosine integral for the function $f(t)=\begin{cases}2t, & 0<t<1\\ 0, & \text{otherwise}\end{cases}$. [102 虎尾車輛 2]

[解]設 $f(t)=\int_0^{\infty} a(\omega)\cos\omega t d\omega$

$$a(\omega)=\frac{2}{\pi}\int_0^{\infty} f(t)\cos\omega t dt=\frac{2}{\pi}\int_0^{1} 2t\cos\omega t dt=\frac{4}{\pi\omega}(t\sin\omega t\Big|_0^1-\int_0^1\sin\omega t dt)$$

$$=\frac{4}{\pi\omega}(\sin\omega+\frac{\cos\omega t}{\omega}\Big|_0^1)=\frac{4}{\pi\omega}(\sin\omega+\frac{\cos\omega-1}{\omega})$$

$$f(t)=\frac{4}{\pi}\int_0^{\infty}(\frac{\sin\omega}{\omega}+\frac{\cos\omega-1}{\omega^2})\cos\omega t d\omega$$

以 $f(t)=\frac{4}{\pi}\int_0^{u}(\frac{\sin\omega}{\omega}+\frac{\cos\omega-1}{\omega^2})\cos\omega t d\omega$ 所繪之圖如下