$f(x)=\begin{cases}k, & -1\le x<1\\ 0, & \text{otherwise}\end{cases}$，(a)求 $f(x)$的傅立葉轉換(Fourier transform) (b)求 $\int_{-\infty}^{\infty}\frac{\sin^2\omega}{\omega^2}d\omega$. [101 虎尾電機 3]

[解](a) $F(\omega)=\int_{-\infty}^{\infty}f(x)e^{-i\omega x}dx=\int_{-1}^{1}ke^{-i\omega x}dx=k\cdot\frac{e^{-i\omega}-e^{i\omega}}{-i\omega}=\frac{2k\sin\omega}{\omega}$

(b)由Parseval's關係式得

$$\int_{-\infty}^{\infty}|f(x)|^2dx=\frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2d\omega \quad\Rightarrow\int_{-1}^{1}k^2\cdot dx=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{2k\sin\omega}{\omega}\right)^2d\omega$$

$$2k^2=\frac{2k^2}{\pi}\int_{-\infty}^{\infty}\frac{\sin^2\omega}{\omega^2}d\omega \quad\Rightarrow\int_{-\infty}^{\infty}\frac{\sin^2\omega}{\omega^2}d\omega=\pi$$