一函數具有週期性：$f(x)=f(x+2\pi)$，其在$-\pi \le x \le \pi$之區間內定義為$f(x)=\begin{cases} 2, & -\pi \le x < 0 \\ 1, & 0 \le x < \pi \end{cases}$. [101 中原土木 5]

[解]$f(x)=\dfrac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$,

$a_0 = \dfrac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx = \dfrac{1}{\pi}(\int_{-\pi}^{0}2dx+\int_{0}^{\pi}1dx)=\dfrac{1}{\pi}(2x\Big|_{-\pi}^{0}+x\Big|_{0}^{\pi})=3$

$a_n = \dfrac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx = \dfrac{1}{\pi}(\int_{-\pi}^{0}2\cos nxdx+\int_{0}^{\pi}\cos nxdx)=\dfrac{1}{\pi}(\dfrac{2\sin nx}{n}\Big|_{-\pi}^{0}+\dfrac{\sin nx}{n}\Big|_{0}^{\pi})=0$

$b_n = \dfrac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx = \dfrac{1}{\pi}(\int_{-\pi}^{0}2\sin nxdx+\int_{0}^{\pi}\sin nxdx)=-\dfrac{1}{\pi}(\dfrac{2\cos nx}{n}\Big|_{-\pi}^{0}+\dfrac{\cos nx}{n}\Big|_{0}^{\pi})$

$=-\dfrac{1}{\pi}[\dfrac{2(1-\cos n\pi)}{n}+\dfrac{\cos n\pi-1}{n}]=\dfrac{\cos n\pi-1}{n\pi}=\dfrac{(-1)^n-1}{n\pi}=-\dfrac{2}{(2n-1)\pi}$

$f(x)=\dfrac{3}{2}-\dfrac{2}{\pi}\sum_{n=1}^{\infty}\dfrac{1}{2n-1}\sin(2n-1)x$