

Expand  $f(x) = x^2$  for  $0 < x < L$ , in a cosine series. [100清大動機7(b)]

$$[\text{解}] f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$$

$$a_0 = \frac{2}{L} \int_0^L f(x) dx = \frac{2}{L} \int_0^L x^2 dx = \frac{2}{L} \cdot x^3 \Big|_0^L = \frac{2}{L} \cdot \frac{L^3}{3} = \frac{2L^2}{3}$$

$$\begin{aligned} a_n &= \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx = \frac{2}{L} \int_0^L x^2 \cos \frac{n\pi x}{L} dx = \frac{2}{L} \cdot \frac{L}{n\pi} \left( x^2 \sin \frac{n\pi x}{L} \Big|_0^L - 2 \int_0^L x \sin \frac{n\pi x}{L} dx \right) \\ &= \frac{2}{n\pi} \left[ 2 \cdot \frac{L}{n\pi} \left( x \cos \frac{n\pi x}{L} \Big|_0^L - \int_0^L \cos \frac{n\pi x}{L} dx \right) \right] = \frac{2}{n\pi} \left( \frac{2L}{n\pi} \cdot L \cos n\pi \right) = \frac{4L^2}{n^2 \pi^2} (-1)^n \end{aligned}$$

$$f(x) = \frac{L^2}{3} + \frac{4L^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos \frac{n\pi x}{L}$$