

Find the Fourier series of the given function $f(x)$, which is assumed to have the period 2π , where $f(x)=0$ if $-\pi < x < 0$, $f(x)=1$ if $0 < x < \pi$. [100 嘉大資工 5]

$$[\text{解}] f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{\pi} \int_0^{\pi} 1 \cdot dx = 1$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nxdx = \frac{1}{\pi} \int_0^{\pi} 1 \cdot \cos nxdx = \frac{1}{n\pi} \cdot \sin nx \Big|_0^{\pi} = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nxdx = \frac{1}{\pi} \int_0^{\pi} 1 \cdot \sin nxdx = -\frac{1}{n\pi} \cdot \cos nx \Big|_0^{\pi} = -\frac{1}{n\pi} (\cos n\pi - 1)$$

$$= -\frac{1}{n\pi} [(-1)^n - 1] = \begin{cases} 0, & n \text{ 為偶數} \\ \frac{2}{n\pi}, & n \text{ 為奇數} \end{cases} = \frac{2}{(2n-1)\pi}$$

$$f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)x$$