Find the Fourier transform of the given function $f(x)=\begin{cases} xe^{-x}, & x>0 \\ 0, & x<0 \end{cases}$. [100 中原機械丙 5]

[解] $\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx = \int_{0}^{\infty} xe^{-x}e^{-i\omega x}dx = \int_{0}^{\infty} xe^{-(1+i\omega)x}dx = -\frac{1}{1+i\omega}[xe^{-(1+i\omega)x}\Big|_{0}^{\infty} - \int_{0}^{\infty} e^{-(1+i\omega)x}dx]$

$$= -\frac{1}{1+i\omega}[0 + \frac{e^{-(1+i\omega)x}}{1+i\omega}\Big|_{0}^{\infty}] = -\frac{0-1}{(1+i\omega)^2} = \frac{1}{(1+i\omega)^2}$$