

Find all possible values of  $(1 + i)^i$ . [99 清大工科乙丙核工甲先進光源 7(b)]

$$\begin{aligned} \text{[解]}(1 + i)^i &= \exp[i \ln(1 + i)] = \exp\{i \ln[\sqrt{2}e^{i(\frac{\pi}{4} + 2k\pi)}]\} = \exp\{i[\ln \sqrt{2} + i(\frac{\pi}{4} + 2k\pi)]\} \\ &= \exp[i \ln \sqrt{2} - (\frac{\pi}{4} + 2k\pi)] = e^{i \ln \sqrt{2}} \cdot e^{-(\frac{\pi}{4} + 2k\pi)} = e^{-(\frac{\pi}{4} + 2k\pi)} (\cos \ln \sqrt{2} + i \sin \ln \sqrt{2}) \end{aligned}$$

其中  $k = 0, \pm 1, \pm 2, \dots$



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