

Find the value of $(1+i)^{2-i}$ in the form $x+iy$. [98 台大工科海洋甲 4]

$$\begin{aligned} \text{[解]} (1+i)^{2-i} &= \exp[(2-i)\ln(1+i)] = \exp\{(2-i)\ln[\sqrt{2}e^{i(\frac{\pi}{4}+2k\pi)}]\} = \exp\{(2-i)[\ln\sqrt{2} + i(\frac{\pi}{4} + 2k\pi)]\} \\ &= \exp[(2\ln\sqrt{2} + \frac{\pi}{4} + 2k\pi) + i(\frac{\pi}{2} + 4k\pi - \ln\sqrt{2})] \\ &= 2\exp(\frac{\pi}{4} + 2k\pi) \cdot \exp i(\frac{\pi}{2} + 4k\pi - \ln\sqrt{2}) = 2\exp(\frac{\pi}{4} + 2k\pi) \cdot \exp i(\frac{\pi}{2} - \ln\sqrt{2}) \\ &= 2\exp(\frac{\pi}{4} + 2k\pi)[\cos(\frac{\pi}{2} - \ln\sqrt{2}) + i\sin(\frac{\pi}{2} - \ln\sqrt{2})] \\ &= 2\exp(\frac{\pi}{4} + 2k\pi)[\sin(\ln\sqrt{2}) + i\cos(\ln\sqrt{2})] \end{aligned}$$

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