

Evaluate the integral $\oint_{\Gamma} \frac{3z^2 \cosh z}{(z+2i)^2} dz$ where Γ is the circle of radius 8 about 1. [98 中央機械能源光機電生醫 4(d)]

[解] $\frac{3z^2 \cosh z}{(z+2i)^2}$ 有二階極點在 $z = -2i$

$$\begin{aligned} R_{-2i} &= \frac{1}{1!} \frac{d}{dz} \left[(z+2i)^2 \cdot \frac{3z^2 \cosh z}{(z+2i)^2} \right] \Bigg|_{z=-2i} = (6z \cosh z + 3z^2 \sinh z) \Bigg|_{z=-2i} \\ &= -12i \cosh(-2i) - 12 \sinh(-2i) = -12[\cosh(-2i) + \sinh(-2i)] = -12e^{-2i} \end{aligned}$$

$$\therefore \oint_{\Gamma} \frac{3z^2 \cosh z}{(z+2i)^2} dz = 2\pi i \cdot R_{-2i} = 2\pi i(-12e^{-2i}) = -24e^{-2i}\pi i$$