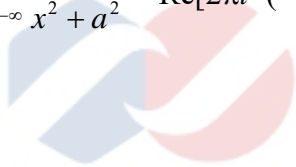


Using the calculus of residues to show  $\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2} dx = \frac{\pi}{a} e^{-a}$ ,  $a > 0$ . [97 海洋光電 10(b)]

[解] 令  $f(z) = \frac{e^{iz}}{z^2 + a^2}$ , 則  $\int_{-\infty}^{\infty} \frac{\cos x dx}{x^2 + a^2}$  為  $\int_{-\infty}^{\infty} f(z) dz$  的實部

$$\text{Res}[f(z); ai] = \left. \frac{e^{iz}}{2z} \right|_{z=ai} = \frac{e^{i(ai)}}{2ai} = -\frac{e^{-a}}{2a} i$$

$$\int_{-\infty}^{\infty} \frac{\cos x dx}{x^2 + a^2} = \text{Re}\left[2\pi i \cdot \left(-\frac{e^{-a}}{2a} i\right)\right] = \frac{\pi}{a} \cdot e^{-a}$$



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