Find the real part of $(1-i)^{1+i}$ if the argument θ is restricted in $0 \leq \theta < 2\pi$. [97 台大機械 6(1)]

[解]$(1-i)^{1+i} = \exp[(1+i)\ln(1-i)] = \exp\{(1+i)[\ln\sqrt{2} + i(-\frac{\pi}{4} + 2k\pi)]\}$

當 $0 \leq \theta < 2\pi$ 時

$(1-i)^{1+i} = \exp[(1+i)(\ln\sqrt{2} + i\frac{7\pi}{4})]$

$= \exp[(\ln\sqrt{2} - \frac{7\pi}{4}) + i(\ln\sqrt{2} + \frac{7\pi}{4})]$

$= \exp(\ln\sqrt{2} - \frac{7\pi}{4}) \cdot \exp[i(\ln\sqrt{2} + \frac{7\pi}{4})]$

$= \sqrt{2}\exp(-\frac{7\pi}{4}) \cdot [\cos(\ln\sqrt{2} + \frac{7\pi}{4}) + i\sin(\ln\sqrt{2} + \frac{7\pi}{4})]$

$(1-i)^{1+i}$ 的實部為 $\sqrt{2}\exp(-\frac{7\pi}{4})\cos(\ln\sqrt{2} + \frac{7\pi}{4})$