Evaluate the integral $\int_0^{\infty} \frac{1}{x^4+1}dx$. [95 高應大機械 8]

[解] $f(z)=\frac{1}{z^4+1}$，極點有 $e^{i\pi/4}, e^{i3\pi/4}, e^{i5\pi/4}, e^{i7\pi/4}$，其中 $e^{i\pi/4}, e^{i3\pi/4}$ 在上半平面

$$\text{Res}[f(z);e^{i\pi/4}]=\lim_{z\to e^{i\pi/4}}\frac{1}{4z^3}=\frac{1}{4e^{i3\pi/4}}=\frac{1}{4(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)}=\frac{-\sqrt{2}-\sqrt{2}i}{2[(-\sqrt{2})^2+(\sqrt{2})^2]}=\frac{-\sqrt{2}-\sqrt{2}i}{8}$$

$$\text{Res}[f(z);e^{i3\pi/4}]=\lim_{z\to e^{i3\pi/4}}\frac{1}{4z^3}=\frac{1}{4e^{i9\pi/4}}=\frac{1}{4(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)}=\frac{\sqrt{2}-\sqrt{2}i}{2[(\sqrt{2})^2+(\sqrt{2})^2]}=\frac{\sqrt{2}-\sqrt{2}i}{8}$$

$$\therefore \int_0^{\infty}\frac{1}{x^4+1}dx=\frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{x^4+1}dx=\frac{1}{2}\cdot 2\pi i\left(\frac{-\sqrt{2}-\sqrt{2}i}{8}+\frac{\sqrt{2}-\sqrt{2}i}{8}\right)=\frac{\sqrt{2}\pi}{4}$$