

Find the solution of the equation $z^2 - (7+i)z + 24 + 7i = 0$. [95 高應大機械 6]

[解] $a = 1, b = -(7+i), c = 24 + 7i$

$$b^2 - 4ac = (49 + 14i + i^2) - 4(24 + 7i) = -48 - 14i = 50\left(-\frac{24}{25} - \frac{7}{25}i\right)$$

$$\text{令 } \cos 2\theta = -\frac{24}{25}, \sin 2\theta = -\frac{7}{25}, x = b^2 - 4ac = 50(\cos 2\theta + i \sin 2\theta)$$

$$\cos \theta = -\sqrt{\frac{1 + \cos 2\theta}{2}} = -\frac{1}{\sqrt{50}}, \sin \theta = \sqrt{\frac{1 - \cos 2\theta}{2}} = \sqrt{\frac{49}{50}} = \frac{7}{\sqrt{50}}$$

$$\sqrt{x} = \sqrt{50}(\cos \theta + i \sin \theta) = -1 + 7i$$

$$z = \frac{-b \pm \sqrt{x}}{2a} = \frac{(7+i) \pm (-1+7i)}{2} = 3 + 4i, 4 - 3i$$

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