Evaluate $\int_{-\infty}^{\infty} \frac{\cos 2x}{1+x^2} dx$. [95 中央機械能源 10(b)]

[解]令 $f(z) = \frac{e^{i2z}}{1+z^2}$，$f(z)$在上半平面有單極點 $z = i$

$$\text{Res}[f(z); i] = \left.\frac{e^{i2z}}{2z}\right|_{z=i} = \frac{e^{-2}}{2i}$$

$$\int_{-\infty}^{\infty} \frac{\cos 2x dx}{1+x^2} = \text{Re}(2\pi i \cdot \frac{e^{-2}}{2i}) = \pi e^{-2}$$