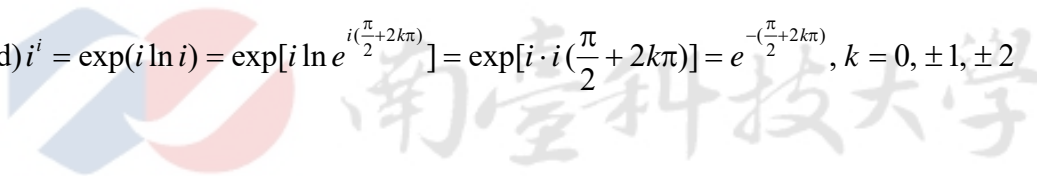Compute the following values, where $i = \sqrt{-1}$: (a)$(1+i)^{20}$ (b)$(\cos\theta + i\sin\theta)^6$, where $\theta = 30°$
(c)$\ln(1+i)$ (d)i^i. [93 清大動機 5]

[解](a)$(1+i)^{20} = [(1+i)^2]^{10} = (2i)^{10} = -1024$

(b)$(\cos 30° + i \sin 30°)^6 = \cos 180° + i \sin 180° = -1$

(c)$\ln(1+i) = \ln(\sqrt{2} e^{i(\frac{\pi}{4}+2k\pi)}) = \ln \sqrt{2} + \ln e^{i(\frac{\pi}{4}+2k\pi)} = \ln \sqrt{2} + i(\frac{\pi}{4} + 2k\pi)$, $k = 0, \pm 1, \pm 2$

(d)$i^i = \exp(i \ln i) = \exp[i \ln e^{i(\frac{\pi}{2}+2k\pi)}] = \exp[i \cdot i(\frac{\pi}{2} + 2k\pi)] = e^{-(\frac{\pi}{2}+2k\pi)}$, $k = 0, \pm 1, \pm 2$