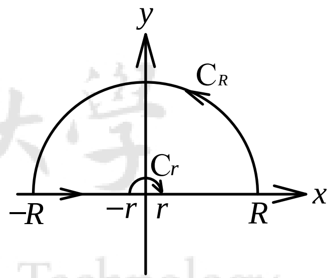
Evaluate $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$. [93 中興材料 4(a)]

[解]令 $f(z) = \frac{e^{iz}}{z}$

$f(z)$有單極點在 $z = 0$，恰在積分路徑上，使用避點圍線



$$\int_{-R}^{-r} \frac{e^{ix}}{x} dx + \int_{C_r} \frac{e^{iz}}{z} dz + \int_{r}^{R} \frac{e^{ix}}{x} dx + \int_{C_R} \frac{e^{iz}}{z} dz = 0$$

$$\lim_{\substack{r\to 0\\ R\to\infty}} \int_{-R}^{-r} \frac{e^{ix}}{x} dx - \pi i \cdot R_0 + \lim_{\substack{r\to 0\\ R\to\infty}} \int_{r}^{R} \frac{e^{ix}}{x} dx + 0 = 0$$

$$\int_{-\infty}^{0} \frac{e^{ix}}{x} dx - \pi i \cdot \frac{e^{i\cdot 0}}{1} + \int_{0}^{\infty} \frac{e^{ix}}{x} dx + 0 = 0$$

$$\int_{-\infty}^{\infty} \frac{e^{ix}}{x} dx = \pi i \Rightarrow \int_{-\infty}^{\infty} \frac{\cos x + i \sin x}{x} dx = \pi i$$

$$\int_{-\infty}^{\infty} \frac{\cos x}{x} dx = 0, \int_{-\infty}^{\infty} \frac{\sin x}{x} dx = \pi$$

而 $\frac{\sin x}{x}$ 為偶函數 $\Rightarrow \int_{0}^{\infty} \frac{\sin x}{x} dx = \frac{1}{2} \int_{-\infty}^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$