

Please evaluate  $\oint_C \frac{3z^2 + 2z - 4}{z^3 - 4z} dz$ , where the contour C is the circle (a)  $|z| = 1$ , (b)  $|z| = 3$ ,

(c)  $|z - 4| = 1$ . [93 中正機械 8]

[解]  $\frac{3z^2 + 2z - 4}{z^3 - 4z} = \frac{3z^2 + 2z - 4}{z(z+2)(z-2)}$

(a)  $|z| = 1$  內部有單極點  $z = 0$

$$R_0 = \left. \frac{3z^2 + 2z - 4}{3z^2 - 4} \right|_{z=0} = 1 \Rightarrow \oint_C \frac{3z^2 + 2z - 4}{z^3 - 4z} dz = 2\pi i$$

(b)  $|z| = 3$  內部有單極點  $z = 0, -2, 2$

$$R_{-2} = \left. \frac{3z^2 + 2z - 4}{3z^2 - 4} \right|_{z=-2} = \frac{12 - 4 - 4}{12 - 4} = \frac{1}{2} \quad R_2 = \left. \frac{3z^2 + 2z - 4}{3z^2 - 4} \right|_{z=2} = \frac{12 + 4 - 4}{12 - 4} = \frac{3}{2}$$

$$\oint_C \frac{3z^2 + 2z - 4}{z^3 - 4z} dz = 2\pi i (R_0 + R_{-2} + R_2) = 6\pi i$$

(c)  $|z - 4| = 1$  內部沒有極點  $\Rightarrow \oint_C \frac{3z^2 + 2z - 4}{z^3 - 4z} dz = 0$