$f(z)=u(x, y)+iv(x, y)$ is an analytic complex function. It is known that $u(x, y)=x^3+3x^2y+axy^2+by^3$. Please find $v(x, y)$ and the values of a and b. [92 中正機械 6]

[解] $v_y = u_x = 3x^2 + 6xy + ay^2 \Rightarrow v = 3x^2y + 3xy^2 + \frac{1}{3}ay^3 + g(x)$

$u_y = -v_x \Rightarrow 3x^2 + 2axy + 3by^2 = -[6xy + 3y^2 + g'(x)]$

比較係數知 $a=-3, b=-1, g'(x) = -3x^2 \Rightarrow g(x) = -x^3$

$v = 3x^2y + 3xy^2 - y^3 - x^3$