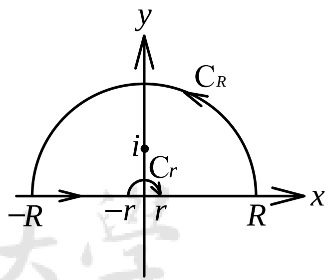
Evaluate the integral $\int_{-\infty}^{\infty} \frac{dx}{x^2 - ix}$. [91 成大造船 8(3)]

[解] $\frac{1}{x^2 - ix} = \frac{1}{x(x-i)}$，令 $f(z) = \frac{1}{z^2 - iz} = \frac{1}{z(z-i)}$



$f(z)$ 有單極點在 $z = 0, i$

$$R_0 = \left.\frac{1}{2z-i}\right|_{z=0} = i \qquad R_i = \left.\frac{1}{2z-i}\right|_{z=i} = -i$$

極點恰在實數軸上，使用避點圍線

$$\int_{-R}^{-r} \frac{dx}{x^2 - ix} + \int_{C_r} \frac{dz}{z^2 - iz} + \int_{r}^{R} \frac{dx}{x^2 - ix} + \int_{C_R} \frac{dz}{z^2 - iz} = 2\pi i \cdot R_i$$

$$\lim_{\substack{r\to 0\\ R\to\infty}} \int_{-R}^{-r} \frac{dx}{x^2 - ix} - \pi i \cdot R_0 + \lim_{\substack{r\to 0\\ R\to\infty}} \int_{r}^{R} \frac{dx}{x^2 - ix} + 0 = 2\pi i \cdot R_i$$

$$\int_{-\infty}^{0} \frac{dx}{x^2 - ix} - \pi i \cdot (i) + \int_{0}^{\infty} \frac{dx}{x^2 - ix} + 0 = 2\pi i \cdot (-i)$$

$$\int_{-\infty}^{\infty} \frac{dx}{x^2 - ix} = \pi$$