

Integrate  $g(z) = \frac{z^2 + 1}{z^2 - \frac{1}{4}}$  counterclockwise around the circle of radius 1 with center (0, 0). [106 淡

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[解]  $g(z) = \frac{z^2 + 1}{(z + \frac{1}{2})(z - \frac{1}{2})} \Rightarrow$  有兩個單極點在  $z = \frac{1}{2}$  及  $z = -\frac{1}{2}$

$$\text{Res}[g(z); \frac{1}{2}] = \left. \frac{z^2 + 1}{2z} \right|_{z=\frac{1}{2}} = \frac{5}{4} \quad \text{Res}[g(z); -\frac{1}{2}] = \left. \frac{z^2 + 1}{2z} \right|_{z=-\frac{1}{2}} = -\frac{5}{4}$$

$$\therefore \oint_C g(z) dz = 2\pi i \left[ \frac{5}{4} + \left(-\frac{5}{4}\right) \right] = 0$$

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