

Please use theory of residues to find the value of $\int_0^{\infty} \frac{2x^2+1}{x^4+5x^2+4} dx$. [106 中興土木丙 3]

[解] $f(z) = \frac{2z^2+1}{z^4+5z^2+4} = \frac{2z^2+1}{(z^2+1)(z^2+4)}$ 有單極點 $z = i, 2i$ 在上半平面

$$\text{Res}[f(z); i] = \left. \frac{2z^2+1}{4z^3+10z} \right|_{z=i} = \frac{-2+1}{-4i+10i} = \frac{i}{6}$$

$$\text{Res}[f(z); 2i] = \left. \frac{2z^2+1}{4z^3+10z} \right|_{z=2i} = \frac{-8+1}{-32i+20i} = -\frac{7i}{12}$$

$$\therefore \int_0^{\infty} \frac{2x^2+1}{x^4+5x^2+4} dx = \frac{1}{2} \int_{-\infty}^{\infty} \frac{2x^2+1}{x^4+5x^2+4} dx = \pi i \left(\frac{i}{6} - \frac{7i}{12} \right) = \frac{5\pi}{12}$$