Using the geometric series , find Laurent expansions for $f(z)=\dfrac{1}{(z-1)(z-2)}$ valid in $|z|<1$ and valid in $|z|>2$. [105 成大水利 2]

[解] $f(z)=\dfrac{1}{(z-1)(z-2)}=\dfrac{1}{z-2}-\dfrac{1}{z-1}$

(1) $|z|<1,\ f(z)=\dfrac{\frac{1}{2}}{\frac{z}{2}-1}-\dfrac{1}{z-1}=-\dfrac{\frac{1}{2}}{1-\frac{z}{2}}+\dfrac{1}{1-z}=-\dfrac{1}{2}[1+\dfrac{z}{2}+(\dfrac{z}{2})^2+\cdots]+(1+z+z^2+\cdots)$

$$=\sum_{n=0}^{\infty}[1-(\frac{1}{2})^{n+1}]z^n$$

(2) $|z|>2,\ f(z)=\dfrac{\frac{1}{z}}{1-\frac{2}{z}}-\dfrac{\frac{1}{z}}{1-\frac{1}{z}}=\dfrac{1}{z}[1+\dfrac{2}{z}+(\dfrac{2}{z})^2+\cdots]-\dfrac{1}{z}[1+\dfrac{1}{z}+(\dfrac{1}{z})^2+\cdots]$

$$=\sum_{n=1}^{\infty}(2^n-1)(\frac{1}{z})^{n+1}$$