

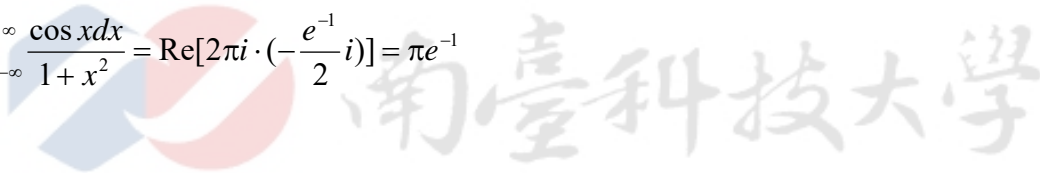
Evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2} dx$. [105 中興精密 4]

[解] 令 $f(z) = \frac{e^{iz}}{1+z^2}$, 則 $\int_{-\infty}^{\infty} \frac{\cos x dx}{1+x^2}$ 為 $\int_{-\infty}^{\infty} f(z) dz$ 的實部

上半平面只有單極點 i

$$\text{Res}[f(z); i] = \left. \frac{e^{iz}}{2z} \right|_{z=i} = \frac{e^{i(i)}}{2i} = -\frac{e^{-1}}{2} i$$

$$\int_{-\infty}^{\infty} \frac{\cos x dx}{1+x^2} = \text{Re}[2\pi i \cdot (-\frac{e^{-1}}{2} i)] = \pi e^{-1}$$



Southern Taiwan University of Science and Technology