

Please evaluate $\oint_C \frac{2z^3 + z^2 + 4}{z^4 + 4z^2} dz$, C the circle $|z-2|=4$, clockwise [105 中山光電 6]

[解] $\frac{2z^3 + z^2 + 4}{z^4 + 4z^2} = \frac{2z^3 + z^2 + 4}{z^2(z^2 + 4)}$ 在 C 內有二階極點 $z = 0$ 、單極點 $z = \pm 2i$

$$R_0 = \frac{1}{1!} \frac{d}{dz} \left[z^2 \cdot \frac{2z^3 + z^2 + 4}{z^2(z^2 + 4)} \right] \Bigg|_{z=0} = \frac{(6z^2 + 2z)(z^2 + 4) - (2z^3 + z^2 + 4)(2z)}{(z^2 + 4)^2} \Bigg|_{z=0} = \frac{1}{4}$$

$$R_{2i} = \frac{2z^3 + z^2 + 4}{4z^3 + 8z} \Bigg|_{z=2i} = \frac{-16i - 4 + 4}{-32i + 16i} = 1$$

$$R_{-2i} = \frac{2z^3 + z^2 + 4}{4z^3 + 8z} \Bigg|_{z=-2i} = \frac{16i - 4 + 4}{32i - 16i} = 1$$

$$\oint_C \frac{2z^3 + z^2 + 4}{z^4 + 4z^2} dz = -2\pi i \cdot \left(\frac{1}{4} + 1 + 1 \right) = -\frac{9\pi i}{2}$$