Is $u=x^2-y^2-y$ a function harmonic? If yes, please find its conjugate harmonic function. [104 交大土木丙 6]

[解] $\frac{\partial u}{\partial x}=2x\Rightarrow\frac{\partial^2 u}{\partial x^2}=2$，$\frac{\partial u}{\partial y}=-2y-1\Rightarrow\frac{\partial^2 u}{\partial y^2}=-2$

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=2+(-2)=0$即$\nabla u=0$，因此$u$ is a harmonic function

設u的conjugate harmonic function為v，則

$v_y=u_x\Rightarrow v_y=2x\Rightarrow v=\int_y 2xdx+f(x)=2xy+f(x)$

又$v_x=-u_y\Rightarrow 2y+f'(x)=-(-2y-1)\Rightarrow f'(x)=1\Rightarrow f(x)=x$

得$v=2xy+x$