

Evaluate $\oint_{\Gamma} \frac{e^{z^3}}{(z-i)^3} dz$, where Γ is a closed path that does not pass through i . [104 中興土木乙
7(2)]

[解](1)若 Γ 內不含 i 時, $\oint_{\Gamma} \frac{e^{z^3}}{(z-i)^3} dz = 0$

(2)若 Γ 內含 i 時, $\frac{e^{z^3}}{(z-i)^3}$ 有三階極點在 $z = i$

$$R_i = \frac{1}{2!} \frac{d^2}{dz^2} \left[(z-i)^3 \cdot \frac{e^{z^3}}{(z-i)^3} \right] \Bigg|_{z=i} = \frac{1}{2} \frac{d}{dz} [3z^2 e^{z^3}] \Bigg|_{z=i} = \frac{1}{2} [(6z + 9z^4) e^{z^3}] \Bigg|_{z=i} = (6i + 9)e^{-i}$$

$$\oint_{\Gamma} \frac{e^{z^3}}{(z-i)^3} dz = 2\pi i \cdot (6i + 9)e^{-i} = (-12 + 18i)\pi e^{-i}$$