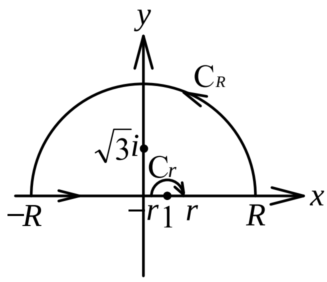
Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x-1)(x^2+3)}$. [104 中興土木乙 7(1)]

[解]令 $f(z)=\frac{1}{(z-1)(z^2+3)}=\frac{1}{z^3-z^2+3z-3}$



$f(z)$有單極點在 $z=1, \sqrt{3}i$

$$R_1=\left.\frac{1}{3z^2-2z+3}\right|_{z=1}=\frac{1}{4}$$

$$R_{\sqrt{3}i}=\left.\frac{1}{3z^2-2z+3}\right|_{z=\sqrt{3}i}=\frac{1}{-6-2\sqrt{3}i}=\frac{-3+\sqrt{3}i}{24}$$

極點恰在實數軸上，使用避點圍線

$$\int_{-R}^{-r}\frac{dx}{(x-1)(x^2+3)}+\int_{C_r}\frac{dz}{(z-1)(z^2+3)}+\int_{r}^{R}\frac{dx}{(x-1)(x^2+3)}$$
$$+\int_{C_R}\frac{dz}{(z-1)(z^2+3)}=2\pi i\cdot R_{\sqrt{3}i}$$

$$\lim_{\substack{r\to 0\\ R\to\infty}}\int_{-R}^{-r}\frac{dx}{(x-1)(x^2+3)}-\pi i\cdot R_1+\lim_{\substack{r\to 0\\ R\to\infty}}\int_{r}^{R}\frac{dx}{(x-1)(x^2+3)}+0=2\pi i\cdot R_{\sqrt{3}i}$$

$$\int_{-\infty}^{0}\frac{dx}{(x-1)(x^2+3)}-\pi i\cdot(\frac{1}{4})+\int_{0}^{\infty}\frac{dx}{(x-1)(x^2+3)}+0=2\pi i\cdot(\frac{-3+\sqrt{3}i}{24})$$

$$\int_{-\infty}^{\infty}\frac{dx}{(x-1)(x^2+3)}=-\frac{\sqrt{3}\pi}{12}$$