$u = \cosh x \cdot \cos y$, find the analytic function $f(z) = u + iv$. [104 中正光電 10]

[解] $v_y = u_x = \sinh x \cdot \cos y \Rightarrow v = \sinh x \cdot \sin y + g(x)$

$u_y = -v_x \Rightarrow -\cosh x \cdot \sin y = -[\cosh x \cdot \sin y + g'(x)] \Rightarrow g(x) = C \Rightarrow v = \sinh x \cdot \sin y$

$f(z) = u + iv = \cosh x \cdot \cos y + i \sinh x \cdot \sin y = \cosh x \cdot \cosh(iy) + i \sinh x \cdot \sinh(iy)$

$= \cosh(x + iy) = \cosh z$