

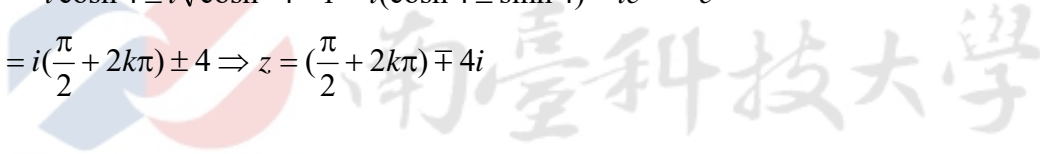
Find all roots of the equation $\sin z = \cosh 4$. [104 中央電機四]

$$[\text{解}] \sin z = \cosh 4 \Rightarrow \frac{e^{iz} - e^{-iz}}{2i} = \cosh 4 \Rightarrow e^{iz} - e^{-iz} = 2i \cdot \cosh 4 \Rightarrow (e^{iz})^2 - 2i \cdot \cosh 4 \cdot e^{iz} - 1 = 0$$

$$e^{iz} = \frac{2i \cdot \cosh 4 \pm \sqrt{(-2i \cdot \cosh 4)^2 + 4}}{2} = \frac{2i \cosh 4 \pm \sqrt{-4 \cosh^2 4 + 4}}{2} = i \cosh 4 \pm \sqrt{1 - \cosh^2 4}$$

$$= i \cosh 4 \pm i \sqrt{\cosh^2 4 - 1} = i(\cosh 4 \pm \sinh 4) = ie^{\pm 4} = e^{i(\frac{\pi}{2} + 2k\pi) \pm 4}$$

$$iz = i\left(\frac{\pi}{2} + 2k\pi\right) \pm 4 \Rightarrow z = \left(\frac{\pi}{2} + 2k\pi\right) \mp 4i$$



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