

Evaluate the complex integral $\oint_C \frac{\sin z}{z^2 + 1} dz$ along the indicated closed contour C_1 , C_2 , and C_3 , respectively. [102 中央機械能源光機電 6]

[解](1) 在 C_1 內無極點 $\Rightarrow \oint_{C_1} \frac{\sin z}{z^2 + 1} dz = 0$

(2) 在 C_2 內只有單極點 $z = -i$

$$R_{-i} = \left. \frac{\sin z}{2z} \right|_{z=-i} = \frac{\sin(-i)}{-2i} = \frac{1}{-2i} \cdot \frac{e^{i(-i)} - e^{-i(-i)}}{2i} = \frac{e - e^{-1}}{4} = \frac{1}{2} \sinh 1$$

$$\oint_{C_2} \frac{\sin z}{z^2 + 1} dz = 2\pi i \left(\frac{1}{2} \sinh 1 \right) = \pi i \sinh 1$$

(3) 在 C_3 內有單極點 $z = i, -i$

$$R_i = \left. \frac{\sin z}{2z} \right|_{z=i} = \frac{\sin(i)}{2i} = \frac{1}{2i} \cdot \frac{e^{i(i)} - e^{-i(i)}}{2i} = \frac{e^{-1} - e^1}{-4} = \frac{1}{2} \sinh 1$$

$$\oint_{C_3} \frac{\sin z}{z^2 + 1} dz = 2\pi i (R_{-i} + R_i) = 2\pi i \sinh 1$$

