Please find the value of $\oint_C \dfrac{z}{(z+1)^2(z+3)}\,dz$, where $z=x+iy$, C is the circle of $|z|=2$. [100 嘉大土木 1]

[解]在 C 內的 pole 只有二階 $z=-1$

$$R_{-1} = \frac{1}{1!}\frac{d}{dz}\left[(z+1)^2\cdot\frac{z}{(z+1)^2(z+3)}\right]\bigg|_{z=-1} = \frac{(z+3)-z}{(z+3)^2}\bigg|_{z=-1} = \frac{3}{4}$$

$$\oint_C \frac{z}{(z+1)^2(z+3)}\,dz = 2\pi i\cdot\frac{3}{4} = \frac{3\pi i}{2}$$