

Please find the value of $\oint_C \frac{z}{(z+1)^2(z+3)} dz$, where $z=x+iy$, C is the circle of $|z|=2$. [100 嘉大土木 1]

[解]在 C 內的 pole 只有二階 $z = -1$

$$R_{-1} = \frac{1}{1!} \frac{d}{dz} \left[(z+1)^2 \cdot \frac{z}{(z+1)^2(z+3)} \right] \Bigg|_{z=-1} = \frac{(z+3) - z}{(z+3)^2} \Bigg|_{z=-1} = \frac{3}{4}$$

$$\oint_C \frac{z}{(z+1)^2(z+3)} dz = 2\pi i \cdot \frac{3}{4} = \frac{3\pi i}{2}$$



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