Find all roots of (a) $\sqrt[3]{1+i}$ (b) $\sqrt[4]{-7+24i}$. [100 中興機械 6]

[解](1)令 $z = \sqrt[3]{1+i}$，而 $1+i = \sqrt{2}e^{i\frac{\pi}{4}}$

$z = [\sqrt{2}e^{i(\frac{\pi}{4}+2k\pi)}]^{1/3} = \sqrt[6]{2}e^{i(\frac{\pi}{4}+2k\pi)/3}$

$k=0$ 時，$z = \sqrt[6]{2}e^{i\frac{\pi}{12}}$ $k=1$ 時，$z = \sqrt[6]{2}e^{i\frac{3\pi}{4}}$ $k=2$ 時，$z = \sqrt[6]{2}e^{i\frac{17\pi}{12}}$

(2)令 $z = \sqrt[4]{-7+24i}$，而 $-7+24i = 25e^{i4\theta}$，其中 $\cos 4\theta = -\dfrac{7}{25}, \sin 4\theta = \dfrac{24}{25}$

$\cos 2\theta = \sqrt{\dfrac{1+\cos 4\theta}{2}} = \dfrac{3}{5}, \sin 2\theta = \sqrt{\dfrac{1-\cos 4\theta}{2}} = \dfrac{4}{5}$

$\cos \theta = \sqrt{\dfrac{1+\cos 2\theta}{2}} = \dfrac{2}{\sqrt{5}}, \sin \theta = \sqrt{\dfrac{1-\cos 2\theta}{2}} = \dfrac{1}{\sqrt{5}}$

$z = \sqrt[4]{25}e^{i(4\theta+2k\pi)/4} = \sqrt{5}e^{i(4\theta+2k\pi)/4}$

$k=0$ 時，$z = \sqrt{5}e^{i\theta} = 2+i$ $k=1$ 時，$z = \sqrt{5}e^{i(\theta+\pi/2)} = -1+2i$

$k=2$ 時，$z = \sqrt{5}e^{i(\theta+\pi)} = -2-i$ $k=3$ 時，$z = \sqrt{5}e^{i(\theta+3\pi/2)} = 1-2i$