Determine the integral $\int_0^{\infty} \frac{x \sin x dx}{(x^2+1)(x^2+4)}$. [100 中央機械能源光機電 10]

[解] $f(z) = \frac{z \sin z}{(z^2+1)(z^2+4)} = \frac{z \sin z}{z^4+5z^2+4}$，有單極點 $z = i, 2i$ 在上半平面

$$R_i = \left.\frac{z \sin z}{4z^3+10z}\right|_{z=i} = \frac{i \sin i}{-4i+10i} = \frac{1}{6}\sin i = \frac{1}{6}\cdot\frac{e^{i(i)}-e^{-i(i)}}{2i} = -\frac{e^{-1}-e}{12} i = \frac{i}{6}\sinh 1$$

$$R_{2i} = \left.\frac{z \sin z}{4z^3+10z}\right|_{z=2i} = \frac{2i \sin 2i}{-32i+20i} = -\frac{1}{6}\sin 2i = -\frac{1}{6}\cdot\frac{e^{i(2i)}-e^{-i(2i)}}{2i} = \frac{e^{-2}-e^{2}}{12} i = -\frac{i}{6}\sinh 2$$

$$\therefore \int_0^{\infty} \frac{x \sin x}{(x^2+1)(x^2+4)} dx = \frac{1}{2}\int_{-\infty}^{\infty} \frac{x \sin x}{(x^2+1)(x^2+4)} dx$$

$$= \pi i \left(\frac{i}{6}\sinh 1 - \frac{i}{6}\sinh 2\right) = \frac{\pi}{6}(\sinh 2 - \sinh 1)$$