已知純量函數$\phi(x, y, z) = 2xz + e^y z^2$；試問此場在點$(1, 0, 1)$沿方向 $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ 之變化率。[97 交大土木丁 12]

[解]$\nabla\phi = \dfrac{\partial\phi}{\partial x}\mathbf{i} + \dfrac{\partial\phi}{\partial y}\mathbf{j} + \dfrac{\partial\phi}{\partial z}\mathbf{k} = 2z\mathbf{i} + e^y z^2\mathbf{j} + (2x + 2ze^y)\mathbf{k} \Rightarrow \nabla\phi\big|_{(1,0,1)} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$

方向$2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$的單位向量為$\mathbf{u} = \dfrac{2\mathbf{i} + 3\mathbf{j} - \mathbf{k}}{\sqrt{2^2 + 3^2 + (-1)^2}} = \dfrac{2\mathbf{i} + 3\mathbf{j} - \mathbf{k}}{\sqrt{14}}$

ϕ在點$(1, 0, 1)$沿方向$2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$的變化率為

$$\nabla\phi\big|_{(1,0,1)} \cdot \mathbf{u} = (2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \cdot \frac{2\mathbf{i} + 3\mathbf{j} - \mathbf{k}}{\sqrt{14}} = \frac{4 + 3 - 4}{\sqrt{14}} = \frac{3}{\sqrt{14}}$$