(a)Show that the vectors $\begin{Bmatrix}1\\1\\0\end{Bmatrix}, \begin{Bmatrix}0\\1\\1\end{Bmatrix}, \begin{Bmatrix}1\\0\\1\end{Bmatrix}$ form a basis for the vector space R^3.

(b)Use the basis given in part (a) to construct an orthonormal basis for the same vector space R^3 with the Gram-Schimdt orthogonalization process. [88 清大動機 3]

[解](a)$\begin{vmatrix}1 & 0 & 1\\1 & 1 & 0\\0 & 1 & 1\end{vmatrix} = 1+1+0-0-0-0 = 2 \neq 0 \Rightarrow$ 三向量為獨立向量，可當基底

(b)令$\boldsymbol{\varphi}_1 = \mathbf{a}_1 = (1, 1, 0), \mathbf{a}_2 = (0, 1, 1), \mathbf{a}_3 = (1, 0, 1) \Rightarrow \mathbf{e}_1 = \dfrac{\boldsymbol{\varphi}_1}{|\boldsymbol{\varphi}_1|} = \dfrac{1}{\sqrt{2}}(1, 1, 0)$

$$\boldsymbol{\varphi}_2 = \mathbf{a}_2 - <\mathbf{a}_2, \mathbf{e}_1> \mathbf{e}_1 = (0, 1, 1) - \frac{0+1+0}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}(1, 1, 0) = (0, 1, 1) - \frac{1}{2}(1, 1, 0) = \frac{1}{2}(-1, 1, 2)$$

$$\mathbf{e}_2 = \frac{\mathbf{b}_2}{|\mathbf{b}_2|} = \frac{1}{\sqrt{6}}(-1, 1, 2)$$

$$\mathbf{b}_3 = \mathbf{a}_3 - <\mathbf{a}_3, \mathbf{e}_1> \mathbf{e}_1 - <\mathbf{a}_3, \mathbf{e}_2> \mathbf{e}_2 = (1, 0, 1) - \frac{1+0+0}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}(1, 1, 0) - \frac{-1+0+2}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}}(-1, 1, 2)$$

$$= (1, 0, 1) - \frac{1}{2}(1, 1, 0) - \frac{1}{6}(-1, 1, 2) = \frac{1}{6}(4, -4, 4)$$

$$\mathbf{e}_3 = \frac{\mathbf{b}_3}{|\mathbf{b}_3|} = \frac{1}{\sqrt{3}}(1, -1, 1)$$

orthonormal basis is $\mathbf{e}_1 = \dfrac{1}{\sqrt{2}}(1, 1, 0), \mathbf{e}_2 = \dfrac{1}{\sqrt{6}}(-1, 1, 2), \mathbf{e}_3 = \dfrac{1}{\sqrt{3}}(1, -1, 1)$