

Solve the differential equation $u_{xx} - u_{xy} - 2u_{yy} = 0$. [84 成大機械 6]

[解] $A = 1, B = -1, C = -2 \Rightarrow B^2 - 4AC = 1 + 8 = 9$

$$\frac{dy}{dx} = \frac{B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-1 \pm 3}{2} = 1, -2$$

$$\text{令 } \xi = x - y, \eta = 2x + y$$

$$u_x = u_\xi + 2u_\eta, u_y = -u_\xi + u_\eta$$

$$u_{xx} = (u_{\xi\xi} + 2u_{\xi\eta}) + 2(u_{\eta\xi} + 2u_{\eta\eta}) = u_{\xi\xi} + 4u_{\xi\eta} + 4u_{\eta\eta}$$

$$u_{xy} = (-u_{\xi\xi} + u_{\xi\eta}) + 2(-u_{\eta\xi} + u_{\eta\eta}) = -u_{\xi\xi} - u_{\xi\eta} + 2u_{\eta\eta}$$

$$u_{yy} = -(-u_{\xi\xi} + u_{\xi\eta}) + (-u_{\eta\xi} + u_{\eta\eta}) = u_{\xi\xi} - 2u_{\xi\eta} + u_{\eta\eta}$$

$$\text{原式} \Rightarrow (u_{\xi\xi} + 4u_{\xi\eta} + 4u_{\eta\eta}) - (-u_{\xi\xi} - u_{\xi\eta} + 2u_{\eta\eta}) - 2(u_{\xi\xi} - 2u_{\xi\eta} + u_{\eta\eta}) = 0 \Rightarrow 9u_{\xi\eta} = 0$$

$$\text{解為 } u = f(\xi) + g(\eta) = f(x - y) + g(2x + y)$$