Find the analytic function $f(z) = u(x, y) + iv(x, y)$, where $v(x, y) = e^{-3x}\sin 3y$. [106 中山光電 10]

[解]$u_x = v_y = 3e^{-3x}\cos 3y \Rightarrow u = \int_x 3e^{-3x}\cos 3y\, dx + g(y) = -e^{-3x}\cos 3y + g(y)$

$u_y = -v_x \Rightarrow 3e^{-3x}\sin 3y + g'(y) = 3e^{-3x}\sin 3y \Rightarrow g'(y) = 0 \Rightarrow \Leftleftarrows g(y) = 0$

$f(z) = -e^{-3x}\cos 3y + ie^{-3x}\sin 3y = -e^{-3x}(\cos 3y + i\sin 3y)$

$\quad\quad = -e^{-3x} \cdot e^{i3y} = -e^{-3(x-iy)} = -e^{-3\bar{z}}$